Universal c-axis conductivity of high-T_c oxides in the superconducting state

T. Xiang
Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing 100080, People's Republic of China

W. N. Hardy
Department of Physics and Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia, V6T 1Z1, Canada

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The anisotropy in the temperature dependence of the in-plane and c-axis conductivities of high-T_c cuprates in the superconducting state is shown to be consistent with a strong in-plane momentum dependence of both the quasiparticle scattering rate and the interlayer hopping integral. Applying the cold spot scattering model recently proposed by Ioffe and Millis to the superconducting state, we find that the c-axis conductivity varies approximately as T^3 in an intermediate temperature regime, in good agreement with the experimental result for optimally doped $YBa_2Cu_3O_{7-x}$ and $Bi_2Sr_2CaCu_2O_{8+x}$.

Microwave surface impedance measurements have provided important information on the pairing symmetry [1] and quasiparticle relaxation [2] in the superconducting state of high-T_c oxides. For $YBa_2Cu_3O_{7-x}$ (YBCO) (and similarly for other high-T_c compounds [3]), the in-plane microwave conductivity σ_{ab} exhibits a large peak centered at approximately 25K in the superconducting state [4–6]. This peak structure of σ_{ab} is due to a competition between the quasiparticles life time and the normal fluid density. From T_c down to 25K the quasiparticle life time increases much more rapidly than the decrease of normal fluid density, causing σ_{ab} to rise with decreasing temperature. At low temperature, the quasiparticle life time reaches a limit and increases very slowly but the normal fluid density continues to fall, σ_{ab} therefore falls with decreasing temperature. Along the c-axis, the conductivity behaves very differently [5]. In contrast to its in-plane counterpart, σ_c falls below T_c and does not show a conductivity peak. In optimally doped YBCO, σ_c rises slightly below 20K. But in $Ba_2Sr_2CaCu_2O_8$ (BSCCO) [7,8], $La_{2-x}Sr_xCuO_4$ [9], $Tl_2Ba_2CuO_6$ and $Tl_2Ba_2CaCu_2O_8$ [10], σ_c drops continuously from T_c down to very low temperature, and no upturn appears.

The different temperature dependences of σ_{ab} and σ_c are not what one might expect within the conventional theory of anisotropic superconductors. In this paper we present a detailed theoretical analysis for the c-axis conductivity. We shall show that the decrease of σ_c immediately below T_c is explained by the fact that the region near the nodes, where the long lived quasiparticles exist, does not enter into the c-axis transport because of the anisotropic interlayer hopping integral.

Let us first consider the behaviour of quasiparticle scattering in high-Tc materials. An important feature revealed by photoemission measurements is that the life time of quasiparticles is long along the Brillouin-zone diagonals and short along other directions on the Fermi surface [11] in both the normal and supercondcuting states. Based on this experimental result and the anisotropic temperature dependence of the in-plane and c-axis resistivity, Ioffe and Millis (IM) [12,13] proposed a cold spot model to account for the normal state transport data. They assumed that the scattering rate of quasiparticles contains a large angular dependent part that vanishes quadratically as the momentum approaches the $(0,0)-(\pi,\pi)$ line with negligible frequency and temperature dependence and an isotropic but temperature dependent part, i.e. $\Gamma_\theta = \Gamma_0 \cos^2 2\theta + \tau^{-1}$, where θ is the angle between the in-plane momentum of the electron and the a-axis. This type of the scattering rate was also used by Hussey *et al.* in the analysis of the angular dependent c-axis magnetoresistance [14]. In Ref. [12], Ioffe and Millis has further assumed that τ^{-1} has the conventional Fermi liquid form $\tau_{FL}^{-1} = \frac{T^2}{T_0} + \tau_{imp}^{-1}$. With this phenomenological model, they gave a good explanation for the temperature dependences of several transport coefficients in the normal state. Van der Marel [15] has recently shown that this model also provides a good description for both the in-plane and c-axis optical conductivities in the normal state.

The cold spot scattering rate, as discussed by IM [12], may be caused by interaction of electrons with nearly singular $d_{x^2-y^2}$ pairing fluctuations. In the superconducting state, as the $d_{x^2-y^2}$ -wave channel scattering is enhanced, the assumption of cold spot scattering made by IM is strengthened. This is indeed consistent with the recent photoemission data measured by Valla *et al* [11]. Thus a detailed comparison between theoretical calculations and experimental measurements for the transport coefficients in the superconducting state provides a crucial test for the cold spot model.

In high-Tc superconductors, the electronic structure is highly anisotropic. In particular, the interlayer hopping integral depends strongly on the in plane momentum of electrons. For tetragonal compounds, the c-axis hopping integral is shown to have the form [16–19,22]

$$t_c = -t_\perp \cos^2(2\theta). \quad (1)$$

This anisotropic interlayer hopping integral is a basic property of high-T_c materials. It results from the hybridization between the bonding O 2p orbitals and virtual Cu 4s orbitals in each CuO_2 plane and holds for all high-T_c cuprates with tetragonal symmetry, independent of the number of CuO_2 layers per unit cell [18,19]. This form of the c-axis hopping integral was first found in the band structure calculations of high-T_c oxides [16,17]. However, as shown in Refs. [18,19], it is valid irrespective of the approximations used in these calculations. For Hg_2BaCuO_4 or other non-body centered tetragonal compounds, $t_\perp$ is approximately independent of θ. For a body-centered tetragonal compound, such as $La_{2-x}Sr_xCuO_4$, $t_\perp$ does depend on θ, but in the vicinity of the gap nodes it is finite. Since in the superconducting state the physical properties are mainly determined by the quasiparticle excitations near the gap nodes, for simplicity we ignore the θ-dependence of $t_\perp$ in the discussion given below.

In YBCO, the CuO planes are dimpled with displacements of O in the c direction. O displacements, together with the CuO chains in YBCO, reduce the crystal symmetry and introduce a finite hybridization between the σ and π bands. This hybridization results in a small but finite t_c along zone diagonals which will change the low temperature behavior of the electromagnetic response functions. However, at not too low temperatures, Eq. (1) is still a good approximation.

The conductivity is determined by the imaginary part of the current-current correlation function. If vertex corrections are ignored [23], it can be shown that the conductivity is given by

$$\sigma_\mu = -\frac{\alpha_\mu}{\pi}\int_{-\infty}^{\infty} d\omega \frac{\partial f(\omega)}{\partial\omega}\int_0^{2\pi}\frac{d\theta}{2\pi}u_\mu^2(\theta)M(\theta) \quad (2)$$

$$M(\theta) = \frac{\pi}{\Gamma_\theta}\mathrm{Re}\frac{(\omega+i\Gamma_\theta)^3-\omega\Delta_0^2\cos^2 2\theta}{\left[(\omega+i\Gamma_\theta)^2-\Delta_0^2\cos^2 2\theta\right]^{3/2}}, \quad (3)$$

where $u_{ab}(\theta) = 1$, $u_c(\theta) = \cos^2(2\theta)$, $\alpha_{ab} = e^2 v_F^2 N(0)/4$, $\alpha_c = e^2 t_\perp^2 N(0)/4$, $N(0)$ is the density of states of electrons at the Fermi level, and $f(\omega)$ is the Fermi function. In obtaining the above equation, the retarded Green's function of the electron, $G_{ret}(k,\omega)$, is assumed to be

$$G_{ret}(k,\omega) = \frac{1}{\omega - \xi_k\tau_3 - \Delta_\theta\tau_1 + i\Gamma_\theta}, \quad (4)$$

where $\xi_k = \varepsilon_{ab}(k) - t_\perp \cos k_z u_c(\theta)$ is the energy dispersion of the electron, $\Delta_\theta = \Delta_0\cos 2\theta$ is the d-wave gap parameter, Γ_θ is the quasiparticle scattering rate, τ_1 and τ_3 are the Pauli matrices.

In the normal state, $\Delta_0 = 0$, and Eq. (2) becomes

$$\sigma_\mu = \alpha_\mu \int_0^{2\pi}\frac{d\theta}{2\pi}\frac{u_\mu^2(\theta)}{\Gamma_\theta}. \quad (5)$$

If the scattering is isotropic, i.e. $\Gamma_\theta = \Gamma(T)$ independent of θ, then σ_{ab} and σ_c should have the same temperature dependence, in contradiction with experiments. However, if $\Gamma_\theta = \Gamma_0\cos^2 2\theta + \tau^{-1}(T)$, then

$$\sigma_{ab} = \frac{\alpha_{ab}}{\sqrt{\tau^{-1}(\Gamma_0+\tau^{-1})}}, \quad (6)$$

$$\sigma_c = \frac{\alpha_c}{2\Gamma_0}\left(1 - \frac{2\tau^{-1}}{\Gamma_0} + \frac{2\tau^{-1}}{\Gamma_0}\sqrt{\frac{\tau^{-1}}{\Gamma_0+\tau^{-1}}}\right). \quad (7)$$

When $\Gamma_0 \gg \tau^{-1}$, σ_{ab} is proportional to $\sqrt{\tau}$ not τ. This is the result first obtained by IM with a Boltzman equation analysis. If τ^{-1}varies quadratically with T as in conventional Fermi liquid theory, the resistivity, i.e. σ_{ab}^{-1}, varies linearly with T. This provides a phenomenological account for the linear resistivity of optimal doped cuprates. σ_c depends on two parameters, α_c/Γ_0 and $\tau\Gamma_0$. In the limit $\Gamma_0 \gg \tau^{-1}$, σ_c depends very weakly on T and extrapolates to a finite value $\alpha_c/(2\Gamma_0)$ at $T = 0K$, in qualitative agreement with the experimental data. These results indicate that the simple cold spot model captures the key features of high-T_c transport properties in the normal state, although its microscopic mechanism is still unclear.

In the superconducting state, Eq. (2) cannot be integrated out analytically. However, in the temperature regime $T_c > T \gg \tau_0^{-1}$, where $\tau_0 = \langle\Gamma_\theta^{-1}\rangle$ is the thermal average of Γ_θ^{-1}, the leading order approximation in Γ_θ is valid and the conductivity is given by [23]

$$\sigma_\mu \approx -\alpha_\mu\int_{-\infty}^{\infty} d\omega\frac{\partial f(\omega)}{\partial\omega}\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{u_\mu^2(\theta)}{\Gamma_\theta}\mathrm{Re}\frac{|\omega|}{\sqrt{\omega^2-\Delta_\theta^2}}. \quad (8)$$

τ_0^{-1} can be estimated from the experimental data of the in-plane microwave conductivity σ_{ab} and the normal fluid density with the generalized Drude formula [2]: $\sigma_{ab} \sim n_{ab}\tau_0$. For optimally doped YBCO [6], τ_0^{-1} is less than $1K$ at low temperatures and increases with increasing temperatures. At $60K$, τ_0^{-1} is about $6K$. Close to T_c, τ_0^{-1} becomes larger but still much less than the temperature. This means that the leading order approximation in Γ_θ is valid in nearly the whole temperature range in which the experimental measurements have been done so far, at least for optimally doped YBCO.

If $\Gamma_\theta = \Gamma(T) = \tau^{-1}(T)$ does not depend on θ, Eq. (8) can be simplified to

$$\sigma_\mu = \frac{e^2 n_\mu(T)\tau}{2m}, \quad (9)$$

where $n_\mu(T)$ is the normal fluid density which decreases with decreasing temperatures. This is nothing but the generalized Drude formula which was first used by Bonn *et al.* in their data analysis for the in-plane microwave conductivity in the superconducting state [23,24]. From

Eq. (9), it is easy to show that the ratio of the in- and out-of-plane conductivities σ_{ab}/σ_c is proportional to the ratio n_{ab}/n_c, i.e. $\sigma_{ab}/\sigma_c = n_{ab}/n_c$. However, this does not agree with experiments, even qualitatively. It implies that the scattering rate must be anisotropic, as mentioned previously.

In the cold spot model, $\Gamma_\theta = \Gamma_0 \cos^2 2\theta + \tau^{-1}(T)$, σ_{ab} and σ_c behave very differently. Eq. (8) now can be approximately written as

$$\sigma_a \approx -\frac{T\tau\alpha_a}{\Delta_0}\int_{-\infty}^{\infty} dx \frac{\partial f(xT)}{\partial x}\frac{|x|}{\sqrt{1+T^2\Gamma_0\tau x^2/\Delta_0^2}}, \quad (10)$$

$$\sigma_c \approx \frac{9\alpha_c\zeta[3]T^3}{2\Gamma_0\Delta_0^3} - \frac{(2\ln 2)\,T\alpha_c}{\tau\Gamma_0^2\Delta_0} + \frac{\alpha_c\sigma_a}{\alpha_a\tau^2\Gamma_0^2}, \quad (11)$$

where $\zeta(3) = 1.202$. In the high temperature limit $\Gamma_0\tau T^2/\Delta_0^2 \gg 1$,

$$\sigma_{ab} \approx \alpha_{ab}\sqrt{\frac{\tau}{\Gamma_0}}, \quad (12)$$

$$\sigma_c \approx \frac{9\alpha_c\zeta(3)T^3}{2\Gamma_0\Delta_0^3} = 9\zeta(3)\sigma_{n,c}(0K)\frac{T^3}{\Delta_0^3}, \quad (13)$$

where $\sigma_{n,c}(0K) = \alpha_c/2\Gamma_0$ is the extrapolated normal state c-axis conductivity at $0K$. These equations reveal a few interesting properties of the conductivities. Firstly, σ_{ab} is proportional to $\sqrt{\tau}$ and does not depend explicitly on Δ_0. This $\sqrt{\tau}$ dependence of σ_{ab} is an extension of Eq. (6) in the superconducting state, which means that σ_{ab} (excluding the fluctuation peak at T_c) will change smoothly across T_c since τ changes continuously at T_c. The temperature dependence of τ in the superconducting state is unknown. But phenomenologically it can be determined from the measured in-plane conductivity via Eq. (12). Secondly, σ_c decreases monotonically with decreasing temperature and behaves approximately as T^3 in the above temperature regime (Δ_0 depends very weakly on temperature except close to T_c). Furthermore σ_c does not depend on τ, which means that this T^3 behavior is *universal* and independent of the impurity scattering provided it is sufficiently weak that the coherent interlayer tunneling dominates. Furthermore, there is no free adjustable parameters in Eq. (13) since both $\sigma_{n,c}(0K)$ and Δ_0 can be determined directly from experiments. This therefore provides a good opportunity to test the cold spot scattering model by comparison with experiments.

In the low temperature limit $\Gamma_0\tau T^2/\Delta_0^2 \ll 1$, Eq. (8) leads to the following results

$$\sigma_{ab} \approx \frac{(2\ln 2)T\tau\alpha_{ab}}{\Delta_0}, \quad (14)$$

$$\sigma_c \approx \frac{675\zeta[5]\alpha_c\tau}{4}\left(\frac{T}{\Delta_0}\right)^5. \quad (15)$$

In this limit both σ_{ab} and σ_c do not depend on Γ_0. This means that the $\Gamma_0\cos^2 2\theta$ term in Γ_θ is not important in this temperature regime. In fact, Eqs. (14) and (15) are just the results of σ_{ab} and σ_c in an isotropic scattering system as given by Eq. (9) since the normal fluid densities n_{ab} and n_c behave as T and T^5 for tetragonal high-T_c compounds at low temperatures, respectively [18–21]. In real materials where impurity scattering is not negligible, as discussed in Refs. [18,19], this T^5 behaviour of the c-axis normal fluid density is fairly unstable and will be replaced by a T^2 law at low temperatures. In this case, the temperature dependence of σ_c will also be changed.

The condition $\Gamma_0\tau T^2/\Delta_0^2 \gg 1$ can be written as $T/\Delta_0 \gg 1/\sqrt{\Gamma_0\tau}$. Since $\tau_0^{-1} > \tau^{-1}$, the condition $T/\Delta_0 \gg 1/\sqrt{\Gamma_0\tau}$ holds if $T/\Delta_0 \gg 1/\sqrt{\Gamma_0\tau_0}$ is satisfied. If we assume $\Gamma_0 \sim 0.15eV$, which is the value used by IM in their analysis of normal state transport coefficients [12], and $\tau_0^{-1} \sim 6K$ as given by the experimental data for YBCO at $60K$, then $1/\sqrt{\Gamma_0\tau_0}$ is estimated to be about 0.06. Thus the condition $\Gamma_0\tau T^2/\Delta_0^2 \gg 1$ holds at least when $T/\Delta_0 \gg 0.06$ for YBCO. Since $\Delta_0 \sim 2T_c$ at low temperatures, therefore Eqs. (12) and (13) are valid when $T/T_c \gg 0.12$ for optimally doped YBCO.

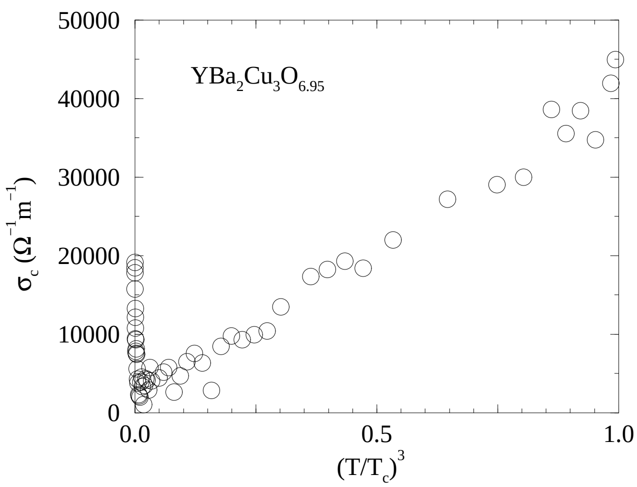

FIG. 1. The c-axis conductivity as a function of $(T/T_c)^3$ for $YBa_2Cu_3O_{6.95}$ measured at 22GHz [5].

To compare the above results with experiments, we plot in Figure 1 the experimental data of σ_c at 22GHz as a function of $(T/T_c)^3$ for $YBa_2Cu_3O_{6.95}$ [5]. From $30K$ up to T_c, σ_c exhibits a T^3 behavior within experimental error, in agreement with Eq. (13). We have fitted the experimental data with other power laws of T/T_c in the same temperature range, but found none of them fit the experimental data as well as the T^3 power law. For this material, there is no experimental data on the temperature dependence of σ_c in the normal state. But just above T_c, $\sigma_c \approx 6.3\times 10^4\Omega^{-1}m^{-1}$ [6]. Since the normal state σ_c depends very weakly on T at low T for optimally doped YBCO [26], we can therefore approximately take this value of σ_c as the extrapolated normal state c-axis conductivity at $0K$, i.e. $\sigma_{n,c}(0K) \approx 6.3\times 10^4\Omega^{-1}m^{-1}$. Substituting this value into Eq. (13), we obtain $\sigma_c \approx 6.8\times 10^5 (T/\Delta_0)^3\Omega^{-1}m^{-1}$.

By fitting this theoretical result with the corresponding experimental data in Figure 1, we find that $\Delta_0/T_c \approx 2.6$. This value of Δ_0/T_c agrees with all other published data for optimally doped YBCO within experimental uncertainty. This agreement indicates that not only the leading temperature dependence but also the absolute values of σ_c predicted by Eq. (13) agrees with experiments.

For YBCO, t_c is small but finite at the gap nodes. This has not been considered in the above discussion and if included, the temperature dependence of σ_c at low temperatures will be changed. What effect leads to the upturn of σ_c at low T remains a puzzle to us. The model presented here is inadequate to address this issue. Perhaps the interplay between the CuO chains and CuO_2 is playing an important role. A possible cause for the upturn of σ_c is the proximity effect between CuO chains and CuO_2 planes. However, a detailed analysis for this is too complicated to carry out at present and at this point we are unable to show explicitly whether this effect alone can lead to the observed upturn in σ_c. Another possibility is that this upturn is due to the onset of coherent tunnelling of the *c*-axis hopping whereas before this upturn the c-axis hopping is incoherent. If this is the case, the assumption made in this paper would fail. However, since the dramatic increase in the quasiparticle lifetime [2,6,25] occurs at a temperature much higher t han the σ_c upturn temperature, we believe that this possibility is unlikely to be relevant.

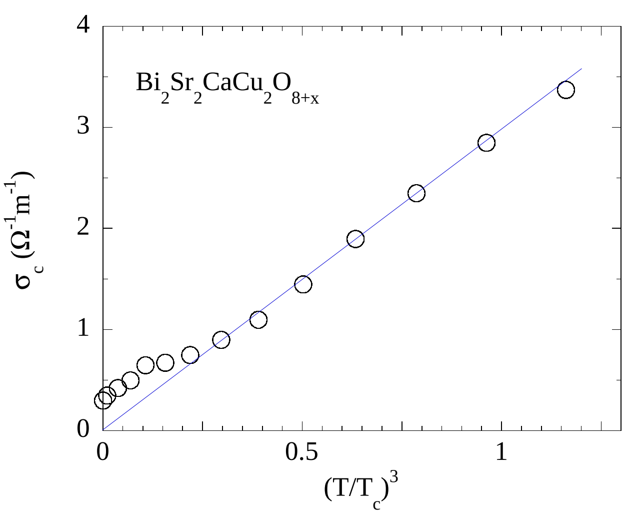

FIG. 2. The c-axis conductivity as a function of $(T/T_c)^3$ for $Bi_2Sr_2CaCu_2O_{8+x}$ ($T_c = 78K$) [7]. The solid line is provided as a guide only.

Figure 2 shows the *c*-axis quasiparticle conductivity obtained by Latyshev *et al.* from an intrinsic mesa tunneling measurement [7] for BSCCO. σ_c of BSCCO also exhibits a T^3 behavior in a broad temperature range in the superconducting state. However, the onset temperature of this T^3 term ($\sim 45K$) is higher than that for YBCO. This is because BSCCO is very anisotropic and disorder effects are stronger than in YBCO. The crossover temperature from the impurity dominated limit at low temperatures to the intrinsic limit at high temperatures, as estimated in Ref. [7], is about 30K. The value of $\sigma_{n,c}(0K)$ for this material is difficult to determine because of the pseudogap effect. Since the opening of a pseudogap always reduces the value of $\sigma_{n,c}$ in the normal state, we can therefore take the value of $\sigma_{n,c}$ just above T_c ($\sim 3\Omega^{-1}m^{-1}$), as a lower bound to $\sigma_{n,c}(0K)$. The c-axis conductivity measured at a voltage well above the pseudogap is nearly temperature independent and higher than the corresponding conductivity in the limit $V \to 0$ (Figure 2 of Ref. [7]). This conductivity, $\sigma_{n,c}(eV > \Delta_0) \sim 8\Omega^{-1}m^{-1}$, sets a upper bound to $\sigma_{n,c}(0K)$. Thus $\sigma_{n,c}(0K)$ is between $3\Omega^{-1}m^{-1}$ and $8\Omega^{-1}m^{-1}$. By fitting the experimental data of σ_c from 45K to T_c with Eq. 13, we find that Δ_0/T_c is within (2.3, 3.1). This range of Δ_0/T_c is consistent with the published data for BSCCO within experimental uncertainty.

In conclusion, we have studied the temperature behavior of microwave conductivity of high-T_c cuprates in the superconducting state within the framework of low energy electromagnetic response theory of superconducting quasiparticles. We found that the c-axis conductivity varies approximately as T^3 and does not depend on $\tau(T)$ in the temperature regime $T_c \gg T \gg \Delta_0/\sqrt{\Gamma_0\tau}$. This *universal* tempearture dependence of σ_c agrees quantitatively with the experimental data for YBCO and BSCCO. Our study shows that it is important to include the anisotropy of the interlayer hopping integral in the analysis of *c*-axis transport properties of high-T_c cuprates.

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